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STRANGE QUARK MATTER ATTACHED TO THE STRING CLOUD IN THE SPHERICAL SYMMETRIC SPACE-TIME ADMITTING CONFORMAL MOTION

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In this paper, we have examined charged strange quark matter attached to the string cloud in the spherical symmetric space-time admitting one-parameter group of conformal motions. For this purpose, we have solved Einstein's field equations for spherical symmetric space-time with strange quark matter attached to the string cloud via conformal motions. Also, we have discussed the features of the obtained solutions.

Keywords: Strange quark matter; string; conformal motion.

1. Introduction

It is still a challenging problem to know the exact physical situation at very early stages of the formation of our universe. At the very early stages of evolution of universe, it is generally assumed that during the phase transition (as the universe passes through its critical temperature) the symmetry of the universe is broken spontaneously. It can give rise to topologically stable defects such as domain walls, strings and monopoles.^{1,2}

Of all these cosmological structures, cosmic strings have excited the most interest. The present day configurations of the universe are not contradicted by the large scale network of strings in the early universe. Moreover, they may act as gravitational lenses and may give rise to density fluctuations leading to the formations of galaxies.³ Strings possess stress energy and are coupled to the gravitational field.

In String Theory, the myriad of particle types is replaced by a single fundamental building block, a 'string'. These strings can be closed, like loops, or open, like a hair. As the string moves through time it traces out a tube or a sheet, according to whether it is closed or open. Furthermore, the string is free to vibrate, and different vibrational modes of the string represent the different particle types, since different modes are seen as different masses or spins.

One mode of vibration, or 'note', makes the string appear as an electron, another as a photon. There is even a mode describing the graviton, the particle carrying the force of gravity, which is an important reason why string theory has received so much attention. The point is that we can make sense of the interaction of two gravitons in string theory in a way we could not in QFT. There are no infinities! And gravity is not something we put in by hand. It has to be there in a theory of strings. So, the first great achievement of string theory was to give a consistent theory of quantum gravity, which resembles GR at macroscopic distances. Moreover string theory also possesses the necessary degrees of freedom to describe the other interactions! At this point a great hope was created that string theory would be able to unify all the known forces and particles together into a single 'Theory of Everything'.

In this study, we will attach strange quark matter to the string cloud. It is plausible to attach strange quark matter to the string cloud. Because, one of such transitions during the phase transitions of the universe could be Quark Gluon Plasma (QGP) \rightarrow hadron gas (called quark-hadron phase transition) when cosmic temperature was $T \sim 200$ MeV.

Recently, Masaharu and Fukutome⁴ and Narodetski *et al.*⁵ have studied tetra quark particle and pentaquarks in the string model, respectively. Also, Harko and Cheng have studied strange quark matter adopting in the form of perfect fluid in the spherical symmetric space-times.⁶

The possibility of the existence of quark matter dates back to early seventies. Bodmer⁷ and Witten⁸ have proposed two ways of formation of strange matter: the quark-hadron phase transition in the early universe and conversion of neutron stars into strange ones at ultrahigh densities. In the theories of strong interaction quark bag models suppose that breaking of physical vacuum takes place inside hadrons. As a result vacuum energy densities inside and outside a hadron become essentially different, and the vacuum pressure on the bag wall equilibrates the pressure of quarks, thus stabilizing the system. If the hypothesis of the quark matter is true, then some of neutrons stars could actually be strange stars, built entirely of strange matter.^{9,10}

Typically, strange quark matter is modeled with an equation of state (EOS) based on the phenomenological bag model of quark matter, in which quark confinement is described by an energy term proportional to the volume.

In this model, quarks are thought as degenerate Fermi gases, which exist only in a region of space endowed with a vacuum energy density B_c (called as the bag constant). Also, in the framework of this model the quark matter is composed of

massless u, d quarks, massive s quarks and electrons.

In the simplified version of the bag model, assuming quarks are massless and non interacting, we then have quark pressure $p_q = \rho_q/3$ (ρ_q is the quark energy density); the total energy density is $\rho = \rho_q + B_c$ but total pressure is $p = p_q - B_c$.

In this paper, we will solve Einstein's field equations for spherical symmetric space-times with strange quark matter attached to the string cloud via Conformal Killing Vector (CKV).

General Relativity provides a rich arena to use symmetries in order to understand the natural relation between geometry and matter furnished by the Einstein equations. Symmetries of geometrical/physical relevant quantities of this theory are known as collineations. The most useful collineations is conformal killing vectors. So, in this paper it is imposed the condition that the space-time manifold admits a conformal killing vector. Conformal Killing Vectors provide a deeper insight into the space-time geometry and facilitate generation of exact solutions to the field equations.

Conformal collineation is defined by,

$$\mathcal{L}_\xi g_{ab} = 2\psi g_{ab} \quad , \quad \psi = \psi(x^a), \quad (1)$$

where \mathcal{L}_ξ signifies the Lie derivative along ξ^a and $\psi(x^a)$ is the conformal factor. In particular, ξ is a special conformal Killing vector (SCKV) if $\psi_{;ab} = 0$ and $\psi_{;a} \neq 0$. Other subcases are homothetic vector (HV) if $\psi_{;a} = 0$ and $\psi \neq 0$, and Killing vector (KV) if $\psi = 0$. Here ; and , denote the covariant and ordinary derivatives, respectively.

The paper is outlined as follows. In section 2, Einstein field equations are obtained for charged strange quark matter attached to the string cloud in the spherical symmetric space-time. In section 3 Einstein field equations are solved for the same matter via conformal motions depending on conformal factor i.e., $\psi(x^a)$. In section 4, concluding remarks are given.

2. Einstein's Field Equations

Let us consider a static distribution of matter represented by charged spherical symmetric matter which may be strange quark matter attached to the string cloud.

In Schwarzschild coordinates the line element takes the following form:

$$ds^2 = e^{\nu(r)} dt^2 - e^{\lambda(r)} dr^2 - r^2 d\Omega^2, \quad (2)$$

with

$$d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2, \quad x^{1,2,3,4} \equiv r, \theta, \phi, t$$

The total energy-momentum tensor T_{ab} is assumed to be the sum of two parts, T_{ab}^S and T_{ab}^E , for string cloud and electromagnetic contributions, respectively, i.e.,

$$T_{ab} = T_{ab}^S + T_{ab}^E. \quad (3)$$

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The energy-momentum tensor for string cloud ¹² is given by

$$T_{ab}^S = \rho U_a U_b - \rho_s X_a X_b \quad (4)$$

here ρ is the rest energy for the cloud of strings with particles attached to them and ρ_s is string tension density; they are related by

$$\rho = \rho_p + \rho_s \quad \text{or} \quad \rho_p = \rho - \rho_s \quad (5)$$

where ρ_p is the particle energy density.

In this paper we will take strange quark matter energy density instead of particle energy density in the string cloud. In this case from Eq. (5), we get

$$\rho = \rho_q + \rho_s + B_c \quad \text{or} \quad \rho_q + B_c = \rho - \rho_s \quad (6)$$

If we put Eq. (6) into Eq. (4), we have for strange quark matter attached to the string cloud

$$T_{ab}^S = (\rho_q + \rho_s + B_c) U_a U_b - \rho_s X_a X_b \quad (7)$$

where U^a is the four velocity $U^a = \delta_4^a e^{-\nu/2}$, X^a is the unit spacelike vector in the radial direction $X^a = \delta_1^a e^{-\lambda/2}$ which represent the strings directions in the cloud, i.e. the direction of anisotropy.

$$T_{ab}^E = -\frac{1}{4\pi} \left(F_a^c F_{bc} - \frac{1}{4} g_{ab} F_{ef} F^{ef} \right) \quad (8)$$

where F_{ab} is the electromagnetic field tensor defined in terms of the four-potential A_a as

$$F_{ab} = A_{b;a} - A_{a;b}.$$

For the electromagnetic field we shall adopt the gauge

$$A_a(0, 0, 0, \phi(r)).$$

Einstein-Maxwell equations can be expressed as

$$R_{ab} - \frac{1}{2} R g_{ab} = 8\pi T_{ab}, \quad (9)$$

$$F_{ab;c} + F_{bc;a} + F_{ca;b} = 0, \quad (10)$$

$$F_{;b}^{ab} = -4\pi J^a, \quad (11)$$

where J^a is the four-current density that becomes $J^a = \bar{\rho}_e U^a$ and $\bar{\rho}_e$ is the proper charge density.

Using the line element (2), the field equations (7)-(11) take the form,

$$8\pi\rho + E^2 = -e^{-\lambda} \left(\frac{1}{r^2} - \frac{\lambda'}{r} \right) + \frac{1}{r^2}, \quad (12)$$

$$-8\pi\rho_s + E^2 = -e^{-\lambda} \left(\frac{1}{r^2} + \frac{\nu'}{r} \right) + \frac{1}{r^2}, \quad (13)$$

$$E^2 = \frac{e^{-\lambda}}{2} \left(\nu'' + \frac{\nu'^2}{2} + \frac{\nu' - \lambda'}{r} - \frac{\nu' \lambda'}{2} \right), \quad (14)$$

$$[r^2 E(r)]' = 4\pi \rho_e r^2. \quad (15)$$

Primes denote differentiation with respect to r , and E is the usual electric field intensity defined as

$$\begin{aligned} F_{41} F^{41} &= -E^2, \\ E(r) &= -e^{-(\nu+\lambda)/2} \phi'(r), \\ \phi'(r) &= F_{14} = -F_{41}. \end{aligned} \quad (16)$$

The charge density ρ_e defined in Eq. (15) is related to the proper charge density $\bar{\rho}_e$ by

$$\rho_e = \bar{\rho}_e e^{\lambda/2}. \quad (17)$$

3. Solutions of the Field Equations

Now we shall assume that space-time admits a one-parameter group of conformal motions (Eq. (1)), i.e.,

$$\mathcal{L}_\xi g_{ab} = \xi_{a;b} + \xi_{b;a} = \psi g_{ab}, \quad (18)$$

where ψ is an arbitrary functions of r . From Eqs. (2) and (18) and by virtue of spherical symmetry, we get the following expressions

$$\xi^1 \nu' = \psi, \quad (19)$$

$$\xi^4 = C_1 = \text{const}, \quad (20)$$

$$\xi^1 = \psi r/2, \quad (21)$$

$$\lambda' \xi^1 + 2\xi_{,1}^1 = \psi, \quad (22)$$

where a comma denotes partial derivatives. From Eqs. (19)-(22), we get

$$e^\nu = C_2^2 r^2, \quad (23)$$

$$e^\lambda = \left(\frac{C_3}{\psi} \right)^2, \quad (24)$$

$$\xi^a = C_1 \delta_4^a + (\psi r/2) \delta_1^a, \quad (25)$$

where C_2 and C_3 are constants of integration.¹¹ Expressions (23)-(25) contain all the implications derived from the existence of the conformal collineation.

Now substituting (23) and (24) into Eqs. (12)-(14), we have

$$\rho + E^2 = (1/r^2)(1 - \psi^2/C_3^2) - 2\psi\psi'/C_3^2 r, \quad (26)$$

$$\rho_s + E^2 = (1/r^2)(1 - 3\psi^2/C_3^2), \quad (27)$$

$$E^2 = \psi^2/C_3^2 r^2 + 2\psi\psi'/C_3^2 r. \quad (28)$$

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Here we use geometrized unit so that $8\pi G = c = 1$. From Eqs. (6) and (26)-(28) we get

$$\rho = \frac{1}{r^2} \left(1 - \frac{2\psi^2}{C_3^2} \right) - \frac{4\psi\psi'}{C_3^2 r}, \quad (29)$$

$$\rho_s = \frac{1}{r^2} \left(1 - \frac{4\psi^2}{C_3^2} \right) - \frac{2\psi\psi'}{C_3^2 r}, \quad (30)$$

$$E^2 = \frac{\psi^2}{C_3^2 r^2} + \frac{2\psi\psi'}{C_3^2 r}, \quad (31)$$

$$\rho_p = \rho_q + B_c = \rho - \rho_s = \frac{2}{r} \left(\frac{\psi^2}{C_3^2} - \frac{\psi\psi'}{C_3^2} \right). \quad (32)$$

Using Eqs. (23) and (24), the line element (Eq. (2)) becomes

$$ds^2 = C_2^2 r^2 dt^2 - \frac{C_3^2}{\psi^2} dr^2 - r^2 d\Omega^2, \quad (33)$$

If the function ψ and an equation of state for the stresses are specified a priori, the problem will be fully determined. So, We will examine the following physically meaningful three cases depending on $\psi(r)$.

Case (i) If $\psi = C_4 r$ then from Eqs. (29)-(32) we get

$$\rho = \rho_s = \frac{1}{r^2} - 6 \left(\frac{C_4}{C_3} \right)^2, \quad (34)$$

$$E^2 = 3 \left(\frac{C_4}{C_3} \right)^2 \quad \text{and} \quad \rho_p = 0. \quad (35)$$

where C_4 is integration constant.

Let us now consider that the charged sphere extends to radius r_0 . Then the solution of Einstein-Maxwell equations for $r > r_0$ is given by the Reissner-Nordström metric as

$$ds^2 = \left(1 - \frac{2M}{r} + \frac{q^2}{r^2} \right) dt^2 - \left(1 - \frac{2M}{r} + \frac{q^2}{r^2} \right)^{-1} dr^2 - r^2 d\Omega^2, \quad (36)$$

and the radial electric field is

$$E = q/r^2, \quad (37)$$

where M and q are the total mass and charge, respectively.

To match the line element (33) with the Reissner-Nordström metric across the boundary $r = r_0$ we require continuity of gravitational potential g_{ab} at $r = r_0$

$$(C_2 r_0)^2 = \left(\frac{\psi}{C_3} \right)^2 = 1 - \frac{2M}{r_0} + \frac{q^2}{r_0^2}, \quad (38)$$

and also we require the continuity of the electric field, which leads to

$$E(r_0) = \frac{q}{r_0^2}, \quad (39)$$

From Eq. (39) and left hand side of Eq. (35) we get

$$\frac{q^2}{r_0^2} = 3r_0^2 \left(\frac{C_4}{C_3} \right)^2, \quad (40)$$

Feeding this expression back into Eq. (38) we obtain

$$\frac{M}{r_0} = \frac{1}{2} + \left(\frac{C_4}{C_3} \right)^2 r_0^2, \quad (41)$$

or from Eqs. (40) and (41) we have

$$M = \frac{r_0}{2} + \frac{1}{3} \frac{q^2}{r_0}, \quad (42)$$

Case (ii) If $\psi = \frac{1}{2} \sqrt{C_3^2 + \frac{4C_5}{r^4}}$, from Eqs. (29)-(32) we get the following expressions

$$\rho = \rho_p = \frac{1}{2r^2} + \frac{6C_5}{C_3^2 r^6} \quad (43)$$

or

$$\rho_q = \frac{1}{2r^2} + \frac{6C_5}{C_3^2 r^6} - B_c, \quad (44)$$

$$E^2 = \frac{1}{4r^2} - \frac{3C_5}{C_3^2 r^6} \quad \text{and} \quad \rho_s = 0. \quad (45)$$

where C_5 is another integration constant.

Setting $\frac{C_5}{C_3^2} = \alpha r_0^4$ and using Eqs. (37), (38) and (45) we obtain the total mass and the total charge:

$$\frac{q^2}{r_0^2} = \frac{(1 - 12\alpha)}{4}, \quad 0 \leq \alpha < \frac{1}{12}$$

$$\frac{M}{r_0} = \frac{1}{2} - 2\alpha$$

Case (iii) If $\psi = \frac{C_6}{\sqrt{r}}$, from Eqs. (29)-(32) we get the following expressions,

$$\rho = \frac{1}{r^2}, \quad (46)$$

$$\rho_s = \frac{1}{r^2} - \frac{3C_6^2}{C_3^2 r^3}, \quad (47)$$

$$\rho_p = \frac{3C_6^2}{C_3^2 r^3} \quad \text{or} \quad \rho_q = \frac{3C_6^2}{C_3^2 r^3} - B_c \quad \text{and} \quad E^2 = 0 \quad (48)$$

where C_6 is a constant.

4. Concluding Remarks

In this paper, we have studied charged strange quark matter attached to the string cloud in the spherical symmetric space-time admitting one-parameter group of conformal motions.

We have obtained the following properties.

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- (a) e^μ and e^λ are positive, continuous and nonsingular for $r < r_0$.
- (b) In the case (i) and case (ii) we have matched our solutions with the Reissner-Nordström metric at $r = r_0$. In the case (i) we have had charged geometric string solutions and charged black string solutions (see Eq. (34) and (42)). In this case, we have obtained the increase of the total mass caused by the charge (see Eq. (42)). Also, if $q = 0$ we get total mass for noncharged black string, i.e. Schwarzschild like black string.
- (c) In the case (ii) and case (iii) we have obtained pressureless charged strange quark matter solution and non charged strange quark matter attached to the string cloud, respectively.
- (d) In the case (ii), the requirement $\rho_p > 0$ throughout the distribution implies that C_5 must be non-negative. However, Eq. (45) gives $E^2 < 0$ in the central region. Thus we have obtained an analytical solution of Einstein-Maxwell equations in the region $r^4 > 12C_5/C_3^2$.
- (e) In the case (iii) from Eqs. (46)-(48) we may conclude that strange quark matter decreases the energy of the string.

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